FREE CONVECTION IN INCLINED LAYERS OF LIQUID AND WITH A JUMPWISE CHANGE IN THE TEMPERATURE OF THE HEAT-TRANSFER SURFACES

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This paper gives the results of investigations of hydrodynamics and heat transfer with the laminar flow of a liquid in inclined layers, closed at the ends, as well as in layers of liquid with a jumpwise change in the temperature of the heat-transfer surface, under conditions of free convection.

Investigations of the hydrodynamics and of the stability of the flow of a liquid in inclined layers with a longitudinal dimension H and a width l, closed at the ends, under thermal-conductivity conditions, are presented in [1, 2]. Values of the heat-transfer coefficients, with different angles of inclination unstably stratified over the thickness of the layer, have been found experimentally [3]. Lykov et al. [4] present investigations of heat transfer at small Rayleigh numbers R, with complex boundary conditions.

It is well known at the present time [5-7] that a transition from thermal-conductivity conditions, under which the vertical temperature gradient β in the mean vertical cross section is equal to 0, to boundary-layer conditions, where $\beta \neq 0$, does not mean the appearance of the instability of laminar flow. With finite dimensions of the relative height of the layer $h = H/l$ and $\beta \neq 0$, with an increase in the R number there is a continuous change in the velocity and temperature profiles, which, at determined values of R, h, and of the Prandtl numbers, lose their stability in a hydrodynamic sense. In inclined layers of liquid with unstable stratification over the thickness of the layer, the flow picture is more complex. The present article is devoted to an experimental investigation of the temperature and velocity profiles under boundary-layer conditions ($\beta \neq 0$) in inclined layers of liquid with stable stratification, when the temperature of the upper heat-transfer surface, t_1, is greater than the temperature of the lower surface, t_2, as well as to an investigation of the hydrodynamics of the flow with a jumpwise change in the temperature of the heat-transfer surfaces.

Let us consider the problem of the flow of a liquid in thin layers, $h \gg 1$, inclined at an angle α with respect to the vertical, with $\beta \neq 0$ and with a constant temperature of the heat-transfer surface. In a cross section with a longitudinal coordinate $z = 0.5h$, and in the neighborhood of a point with a dimensionless coordinate x, normal to the heat-transfer surfaces and equal to 0.5, by virtue of symmetry the horizontal component of the velocity $v = 0$, and the derivative with respect to the vertical component $\partial w/\partial z = 0$.

In this case, we have the following system of equations:

$$R\theta\cos\alpha = B\frac{\partial p}{\partial z} - P^{1/2}\frac{\partial^2 w}{\partial x^2}, \qquad R\theta\sin\alpha = -B\frac{\partial p}{\partial x}, \quad P^{1/2}w\frac{\partial\theta}{\partial z} = \frac{\partial^2\theta}{\partial x^2}, \quad B = \frac{gl^3}{a\nu} \tag{1}$$

Here θ is the dimensionless temperature; p is the pressure; g is the acceleration due to gravity; a is the coefficient of thermal diffusivity, and ν is the coefficient of kinematic viscosity. As units of length, temperature, pressure, and velocity we adopt the following quantities:

$$l,\ t_1 - t_2,\ \rho_0 g l,\ (a\nu)^{1/2} / l$$

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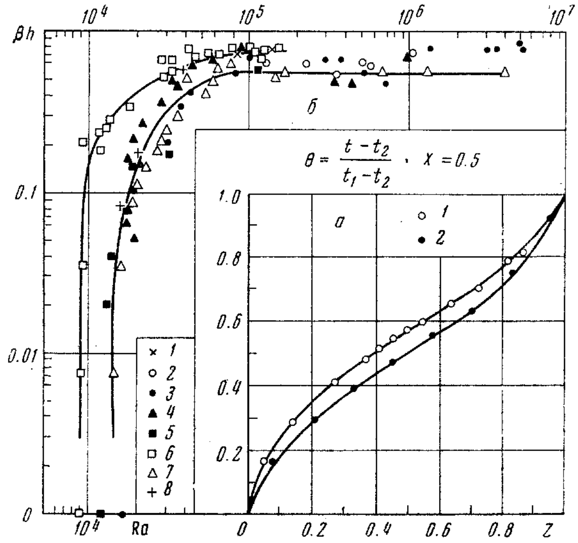

Fig. 1

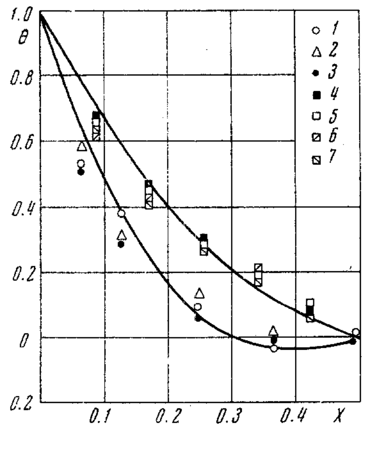

Fig. 2

Under steady-state conditions, Eq. (1) must satisfy the boundary conditions $w = 0$, $\theta = 1$ at $x = 0$; $w = 0$, $\theta = 0$ at $x = 1$, and $w = 0$ at $x = 0.5$.

From the system of equations (1) it follows that $\beta = \text{const}$ and

$$(D^4 + 4m^4)w = 0 \tag{2}$$

where

$$D = \partial/\partial x, \; 4m^4 = \beta R \cos\alpha$$

A solution of Eq. (2) in the approximate theory of the boundary layer [7] can be obtained in the following form:

$$w = -\frac{R\cos\alpha}{4P^{1/2}\, m^2 (f_2 - 1)} [f_1 \cos mx \,\mathrm{sh}\, mx + f_2 \exp mx \sin mx + \exp(-mx)\sin mx] \tag{3}$$

$$\theta = \frac{t - t_{x=0.5}}{t_1 - t_{x=0.5}} = -\frac{1}{f_2 - 1}[f_2 \sin mx \,\mathrm{ch}\, mx - f_2 \exp mx \cos mx + \exp(-mx)\cos mx] \tag{4}$$

where

$$f_1 = -\frac{f_2 \sin {}^1/_2 m \exp {}^1/_2 m + \sin {}^1/_2 m \exp(-{}^1/_2 m)}{\cos {}^1/_2 m \,\mathrm{sh}\, {}^1/_2 m}$$

$$f_2 = \frac{\exp(-{}^1/_2 m)\sin {}^1/_2 m \cos m \,\mathrm{sh}\, m - \exp(-m)\sin m \cos {}^1/_2 m \,\mathrm{sh}\, {}^1/_2 m}{\exp m \sin m \cos {}^1/_2 m \,\mathrm{sh}\, {}^1/_2 m - \exp {}^1/_2 m \sin {}^1/_2 m \cos m \,\mathrm{sh}\, m}$$

The problem consists in the experimental verification of the solutions obtained (3) and (4), as well as in determining the conditions R, P, h, α under which $\beta = \text{const}$.

The experimental unit consists of two flat heat exchangers, having different surface temperatures ($t_1 > t_2$).

The heat exchangers are made of copper and have the dimensions $25 \times 55 \times 395$ mm^3. A constant value of the temperatures of the heat-transfer surfaces was ensured by circulation of thermostatted water in the cavities of the heat exchanger. The temperature of the heat-transfer surfaces was measured using a Nichrome–constantan thermocouple with a diameter of 0.2 mm. The thermocouples were drawn through openings with a diameter of 1.5, arranged along the heat-transfer surface at a distance of 0.5 mm apart, and then, through openings 0.5 mm in diameter, were caulked into the surface.

The emf of the thermocouples was measured using a R-306 low-ohmic direct-current potentiometer in conjunction with an M-195/1 galvanometer. The temperature gradient at the heat-transfer surface was negligibly small.

The plates were placed in a container with a transparent front wall. That the heat-transfer surfaces were parallel was ensured by a frame, calibrated for thickness, which was installed between them. The

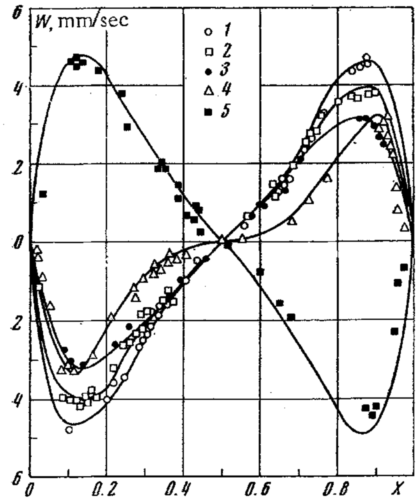

Fig. 3

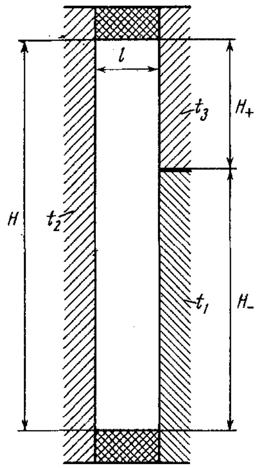

Fig. 4

experimental volume was bounded by the heat-transfer surfaces, the calibrated frame made of Plexiglas, and the glass wall of the container. The container with the heat exchangers was installed in a device which permitted changing the angle of inclination of the layer with respect to the vertical. A positive angle of inclination was taken as one when the heating was effected from the top. The working liquids were water, 96% ethyl alcohol, and 97% glycerin.

The lower limit of the validity of the solutions (3) and (4) can be determined from the change in the value of β with different values of R, α. The Rayleigh number at which $\beta \to 0$ will be the limit of the validity of the solutions (3), (4).

The temperature in the inclined layers of liquid was measured using a copper–constantan temperature probe, made of thermocouple leads with a diameter of 0.06 mm. The thermocouple was led upward through the layer near the vertical face through a capillary with a diameter of 0.2 mm, then, to a distance of 12 mm into the depths of the layer along a capillary with a diameter of 0.5 mm; then, at a depth of 15 mm, the free end of the thermocouple was attached.

The thermocouple probe was introduced into the upper face of the layer through an opening in the calibrated frame, and was led along the lateral face of the layer. The construction of the probe permitted determining the temperature at any given point of one vertical cross section. The coordinates of the position of the thermocouple were determined using a microscope with a fivefold magnification. The temperatures were measured at different transverse cross sections over the height of the layer, for a given Rayleigh number, and for a given angle of inclination, α. The measured temperature field in the layer was used to construct the dependence $\theta = f(z)$ for the cross section $x = 0.5$.

Figure 1a gives the dependences $\theta = f(z)$ in the cross section $x = 0.5$, for different angle of inclination of a layer of ethyl alcohol. Points 1 were obtained in a layer with $\alpha = 30°$, $R = 5.25 \cdot 10^4$, $l = 5.9$ mm, $h = 37.5$, and points 2 in a layer with $\alpha = 70°$, $R = 3.1 \cdot 10^5$, $l = 8.2$ mm, $h = 19.5$.

As is evident from Fig. 1a, there is a constant temperature gradient over the height in the central part of the layer, at different angles of inclination.

Figure 1b gives experimental values of the temperature gradients over the height of the layer in the cross section $x = 0.5$, at different angles of inclination, dimensions of the layer, and P and R numbers. Ethyl alcohol, $P = 14$, is represented by the points 1) $\alpha = 30°$, $h = 37.5$; 2) $\alpha = 50°$, $h = 19.4$; 3) $\alpha = 70°$, $h = 10$-35; 4) $\alpha = 0$, $h = 18$-55. Water, with $P = 6$, is represented by the experimental points: 5) $\alpha = 70°$, $h = 37.5$; 6) $\alpha = 0$, $h = 8$-38. Points 7 denote experimental data for paraffin with $P = 10^5$, $\alpha = 0$ [6]. Glycerin, $P = 10^3$, $\alpha = 60°$, $h = 13$-17, corresponds to point 8.

It follows from Fig. 1b that, at $P = 14$-10^3, the limit of thermal conductivity conditions ($\beta = 0$) does not depend on the angle of inclination and corresponds to the value $R = 1.3$-$1.6 \cdot 10^4$. At $P \sim 6$, there is observed a dependence of thermal-conductivity conditions on the angle of inclination. If at $\alpha = 0$, $R = 9 \cdot 10^3$, at an angle of inclination $\alpha = 70°$, $R = 1.3 \cdot 10^4$.

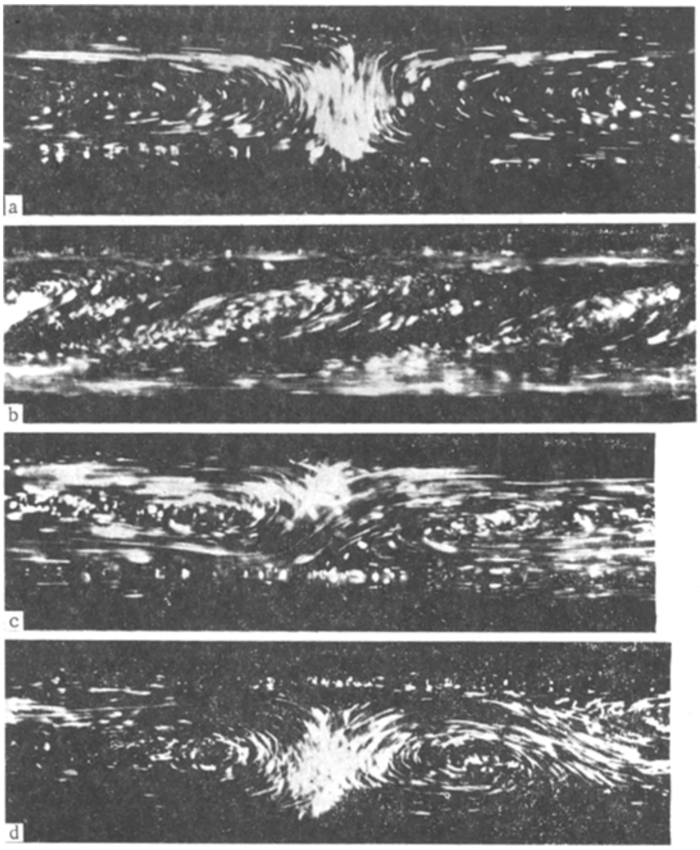

Fig. 5

With the appearance of instability of the flow, there is an increase in βh up to a value of $\beta h \sim 0.7$–0.8 for a liquid with $P \sim 6$, as well as with large angles of inclination, when there develops instability of the type of a longitudinal roller.

Figure 2 compares the experimental temperature profiles with dependence (4). In a layer with $\alpha = 70°$, $R = 3.1 \cdot 10^5$, $l = 8.2$ mm, $H = 160$ mm, measurements were made, at different heights, of the temperature profile $\theta = (t - t_{x=0.5})/(t_1 - t_{x=0.5})$. They are designated by the points: 1) $z = 52.5$ mm; 2) $z = 72.5$ mm; 3) $z = 92.5$ mm. With $\alpha = 30°$, $R = 5.25 \cdot 10^5$, $H = 220$ mm, $l = 5.9$ mm, the measurements correspond to the points: 4) $z = 90$ mm; 5) $z = 100$ mm; 6) $z = 110$ mm; 7) $z = 120$ mm. The solid lines correspond to the theoretical solution of (4) for the above conditions. As is evident from the figure, there is satisfactory agreement between the experimental and theoretical temperature profiles for a large part of the height of the layer, at different angles of inclination.

The rate of flow of the liquid in the layer is determined by recording the trajectories of the motion of suspended particles with a photographic camera. Aluminum particles with a diameter of 5-15 μ are put into the layer for visualization of the flow. The exposure time of the camera was periodically calibrated. The pictures of the flow were taken at a distance of $\sim 3\ l$ from the vertical end of the layer, which made it possible to avoid errors due to end effects.

Figure 3 shows velocity profiles at different angles of inclination. The results of calculation using (3) are shown by the curves, while the results of experimental investigations for a layer with $l = 8.2$ mm, $h = 19.5$ are shown by the points: 1) $\alpha = 30°$, $R = 2.4 \cdot 10^5$; 2) $\alpha = 50°$, $R = 2.4 \cdot 10^5$; 3) $\alpha = 70°$, $R = 3.15 \cdot 10^5$.

As is evident from Fig. 3, there is good agreement between the results of experiment and of calculation using formula (3).

Experimental experiments of natural convection in vertical layers of liquid ($\alpha = 0$) with a jumpwise change in the temperature drop over the height of the layer were carried out in a unit consisting of three flat heat exchangers; a schematic diagram of the unit is shown in Fig. 4. Each of the heat exchangers was

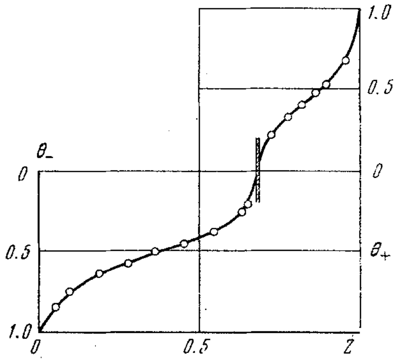

Fig. 6

made of two copper plates, so welded that between them there was a slit measuring 7×60 mm^2. Thermostated water was circulated in the slit of the heat exchanger. Heat exchangers 1, 2, 3 had, respectively, the following dimensions: $27\times 80\times 160$ mm^3, $27\times 80\times 250$ mm^3, $27\times 80\times 90$ mm^3. Heat exchanger 3 was installed on top of heat exchanger 1 in such a way that they made up a single flat heat-transfer surface. The plates were separated by a thermally insulating plate with a thickness of 1 mm. Thus, in a layer of liquid closed at the ends, conditions were created under which the temperature of one heat-transfer surface was constant over the height (t_2), while the temperature of the other varied gradually (t_1, t_3). The experimental method has been described above.

The investigations of the hydrodynamics of the flow were carried out for different types of temperature jumps: $t_3 > t_2 > t_1$, $t_1 > t_2 > t_3$, $t_2 > t_1 > t_3$, $t_2 > t_3 > t_1$.

With a temperature jump of the type $t_3 > t_2 > t_1$, over the height of the temperature jump conditions of stable stratification are created and the flows are directed in opposing directions. In this case, over the height of the temperature jump there is no convective mixing, and two mutually independent flows are formed, above and below the temperature jump.

Figure 5a shows a picture of the flow of the liquid in the region of a temperature jump for a layer with $l = 5.9$ mm, $h_+ = 12$, $R_+ = 8.3\cdot 10^4$, $h_- = 25.5$, $R_- = 8\cdot 10^4$.

Figure 6 gives the profile of the temperature $\theta = (t-t_2)/(t_1-t_2)$ for a layer below a temperature jump, and $\theta_+ = (t-t_2)/(t_3-t_2)$ for the upper layer; this profile shows that the character of the temperature change over the height of layers above and below a temperature jump is exactly the same as in a layer with a constant temperature difference of the heat-transfer surfaces over the height; there is a region with a constant temperature gradient in the layers below and above the jump; the quantity βh, calculated independently for the upper and lower flows, is equal to ~ 0.5. The measurements were made in a layer of ethyl alcohol with $l = 8$ mm, $h_- = 18.7$, $R_- = 1.9\cdot 10^5$, $h_+ = 8.7$, $R_+ = 2.08\cdot 10^5$ in the cross section $x = 0.5$.

The investigations of the hydrodynamics of the flow of the liquid showed that the velocity profiles in the cross section $0.5\ h_+$ for the upper layer (above the temperature jump) and in the cross section $0.5\ h_-$ for the lower layer (below the temperature jump) correspond to the solution of (3). In Fig. 3, the points 4 show the experimental values of the velocity in the upper part of a layer with $l = 8$ mm, $h_+ = 8.7$, $R_+ = 1.95\cdot 10^5$; points 5 refer to the layer below the jump with $h_- = 18.7$, $R_- = 1.9\cdot 10^5$.

These same curves give plots of the theoretical profiles of the velocity (3), valid for a separate layer of liquid with $\Delta t = \text{const}$ over the height.

As is evident from Fig. 3, there is good agreement between the theoretical and experimental profiles.

As in a layer with $\Delta t = \text{const}$, with corresponding dimensions and R numbers, cellular-type flows appear in the upper and lower layers (Fig. 5b).

Figure 5b shows a picture of the flow at the center of the lower part of a layer with $l = 8$ mm, $h_- = 18.6$, $R_- = 8.8\cdot 10^5$.

Thus, with a temperature distribution of the type $t_3 > t_2 > t_1$ there are independent flows above and below a temperature jump.

With the temperature jump $t_1 > t_2 > t_3$ there are also opposing flows; however, in the contrary case $t_3 > t_2 > t_1$, in the region of the temperature jump there exist unstable stratification conditions ($\partial t/\partial z < 0$). Unstable flows arise at the height of the temperature jump. As can be seen in Fig. 5c, in the region of a temperature jump there are flows similar to interwoven jets. The photograph shows the region of a temperature jump in a layer with the parameters $l = 5.9$ mm, $h_- = 25.5$, $R_- = 2.4\cdot 10^4$, $h_+ = 11.9$, $R_+ = 7.8\cdot 10^4$. With an increase in the R number, the flow becomes ever more complex (Fig. 5d) and with $R_+ = 2.66\cdot 10^5$ and there is intense mixing at the height of the temperature jump.

Figure 5d shows the region of a temperature jump with the above values of R in the upper and lower layers and with the parameters $l = 8$ mm, $h_- = 18.7$, $h_+ = 8.75$.

With temperature jumps of the types $t_2 > t_1 > t_3$, $t_2 > t_3 > t_1$ there is an overall upward flow at one surface and a descending flow at the other. The presence of a temperature jump manifests itself in the earlier development of instability of the type of a running wave.

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